UNIVARIATE GAUSSIAN CLASSIFIER

- Some notation; We have a feature x and a set $\{\omega_1, \omega_2, \dots, \omega_J\}$ of J classes. In the univariate case, x is one dimensional, $x \in \mathbb{R}$.

- According to Bayes rule, the posterior probability for class j can be computed as

$$P(\omega_j | x) = \frac{p(x | \omega_j) P(\omega_j)}{p(x)}$$

where $p(x) = \sum_{j=1}^{J} p(x|\omega_j) P(\omega_j)$ and $P(\omega_j)$ is the apriori probability.

A priori probability means the probability given in advance. For example if we have two classes ~~and~~, ω_1 and ω_2 and we know that it is a 75% chance of getting class ω_1 and 25% chance of getting class ω_2 we have $P(\omega_1) = 0,75$ and $P(\omega_2) = 0,25$.

- We want to classify a feature x to the class with the highest posterior probability. That is:

Decide ω_i if $P(\omega_i|x) \geq P(\omega_j|x)$

for all $j \neq i$

- This can be written as; classify x as ω_i

if $$g_i(x) \geq g_j(x),$$ ~~[illegible]~~

where possible discriminant functions are:

$$g_i(x) = P(\omega_i|x) = \frac{p(x|\omega_i)P(\omega_i)}{p(x)}$$

or $$g_i(x) = p(x|\omega_i)P(\omega_i)$$

or $$g_i(x) = \ln(p(x|\omega_i)P(\omega_i)).$$

~~[illegible]~~

All these discriminant functions will have the maximum for ~~[illegible]~~ the same x.

Lets chose to use

This is what you need to implement!

$$g_i(x) = p(x \mid \omega_i) P(\omega_i)$$

and lets use the univariate Gaussian distribution

$$p(x \mid \omega_i) = \frac{1}{\sqrt{2\pi}\,\sigma_i} e^{-\frac{1}{2}\left(\frac{x-\mu_i}{\sigma_i}\right)^2}$$

Where μ_i is the mean for class i and σ_i is the standard deviation for class i.

So, every class will have a mean value μ_i and a variance σ_i^2 estimated from the training data, giving every class a probability distribution $p(x \mid \omega_i)$. To classify a feature value x choose the class with the highest posterior probability $g_i(x) = p(x \mid \omega_i) P(\omega_i)$.