Classification 3

Ex 1a)

$$\Sigma_1 = \begin{bmatrix} 0{,}5 & 0 \\ 0 & 2 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\mu_1 = [3 \quad 6]^T \qquad \mu_2 = [3 \quad -2]^T$$

Finding the eigenvalues of Σ_1:

$$\det \begin{bmatrix} 0{,}5-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} = (0{,}5-\lambda)(2-\lambda)$$

So the eigenvalues is $\lambda_1^1 = 0{,}5$, $\lambda_2^1 = 2$.

And the eigenvectors must be $v_1^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v_2^1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

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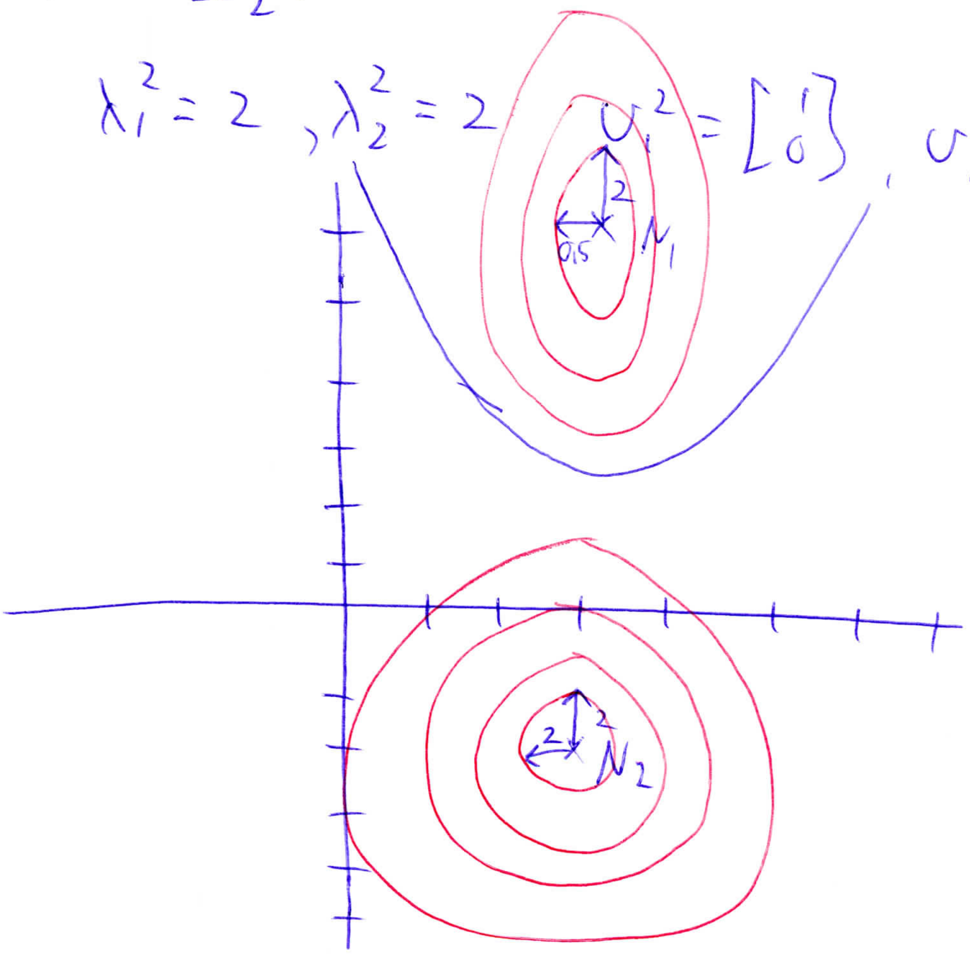

Classification 3

Ex 1 b) Show that $g_1(x) = g_2(x)$ in the case with features x_1 and x_2 can be expressed as:

$$x_2 = 0{,}1875x_1^2 - 1{,}125x_1 + 3{,}5142$$

We have the general case, so $g_j(\bar{x}) = -\frac{1}{2}(\bar{x}-\bar{\mu}_j)^T \Sigma_j^{-1}(\bar{x}-\mu_j) - \frac{1}{2}\ln|\Sigma| + \underbrace{\ln P_{\omega_j}}_{0}$

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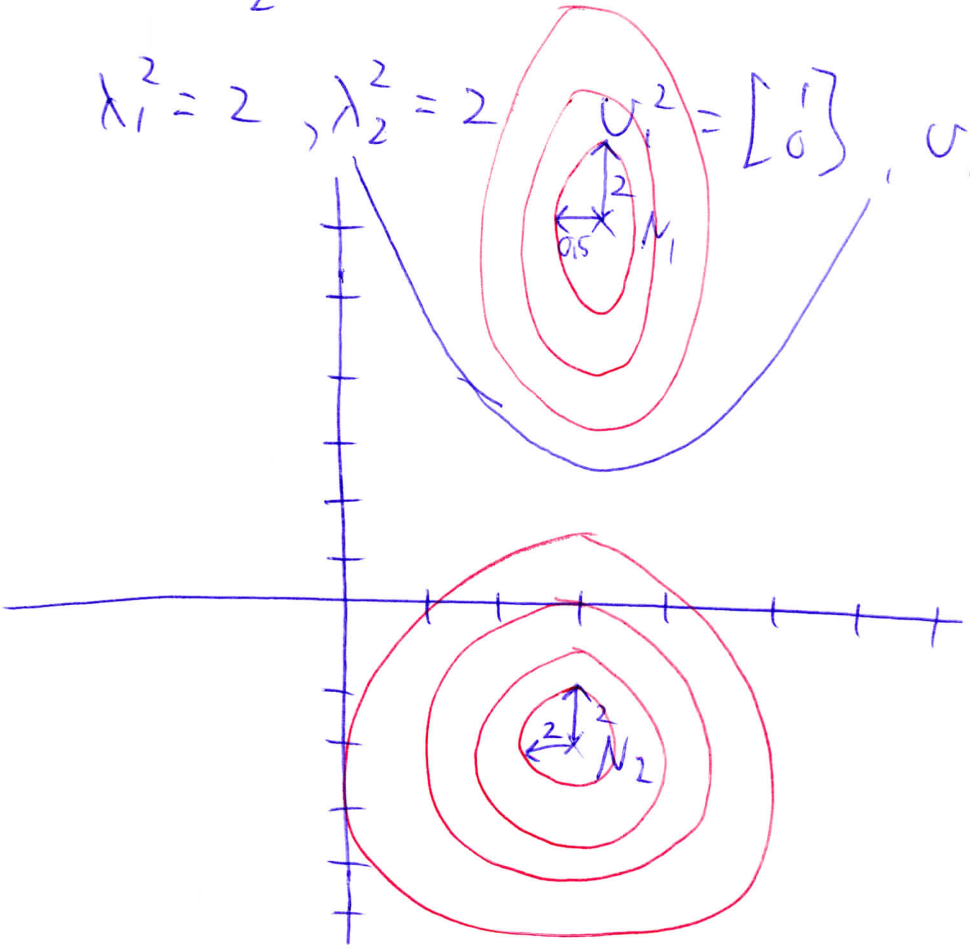

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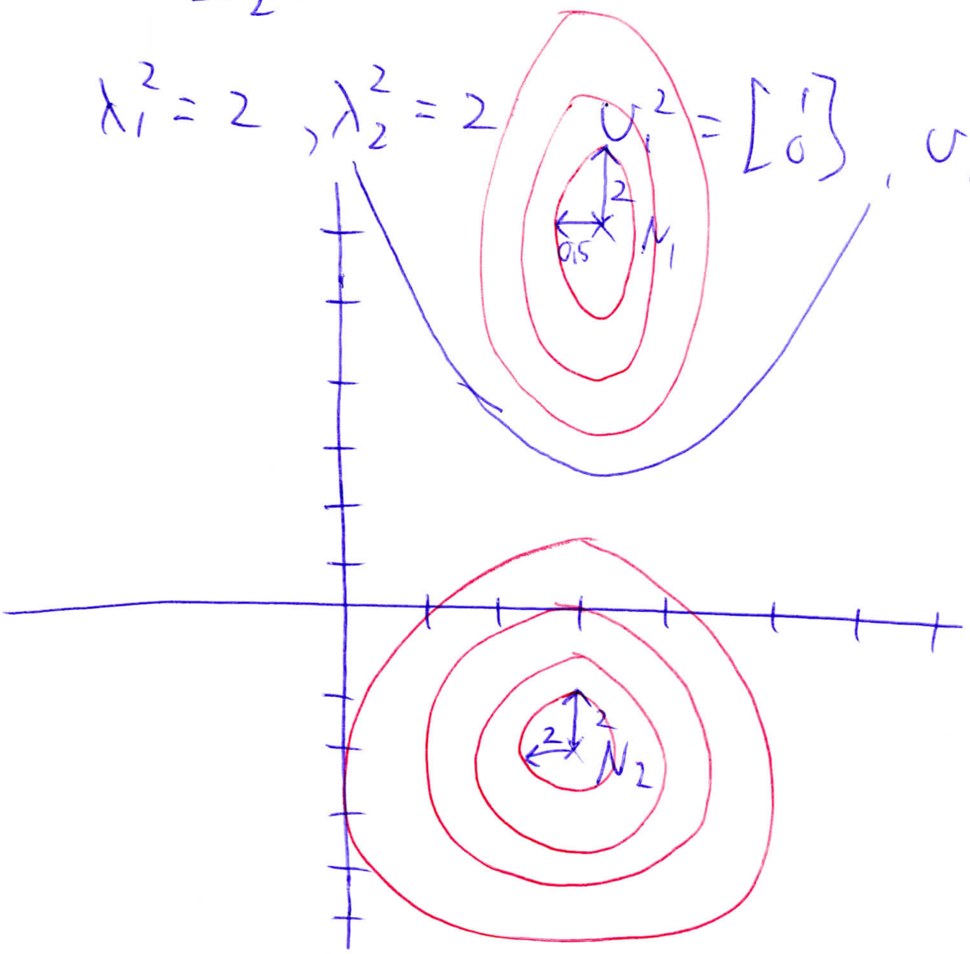

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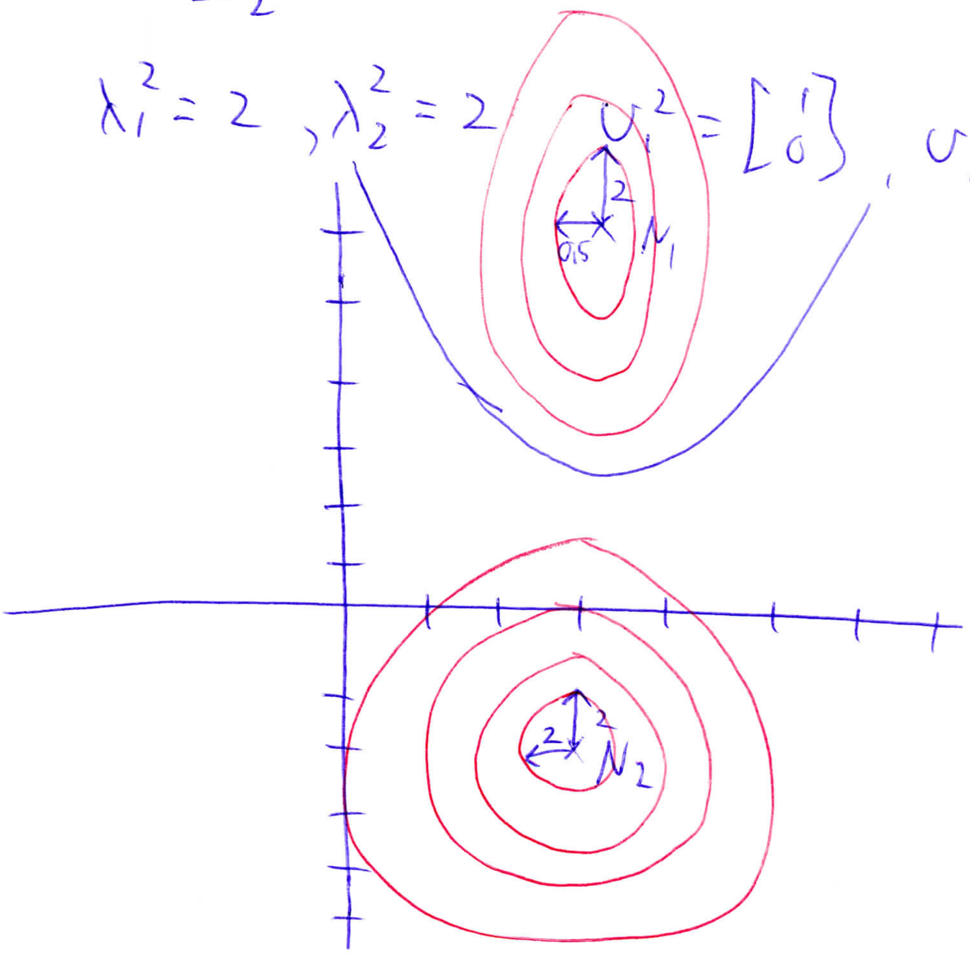

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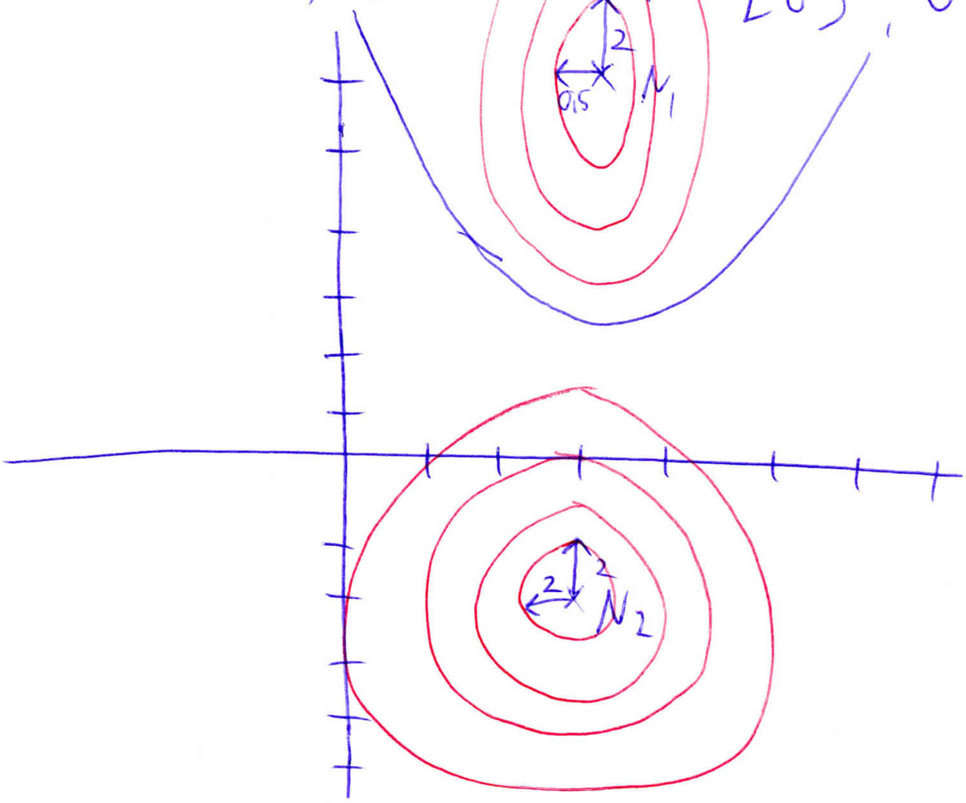

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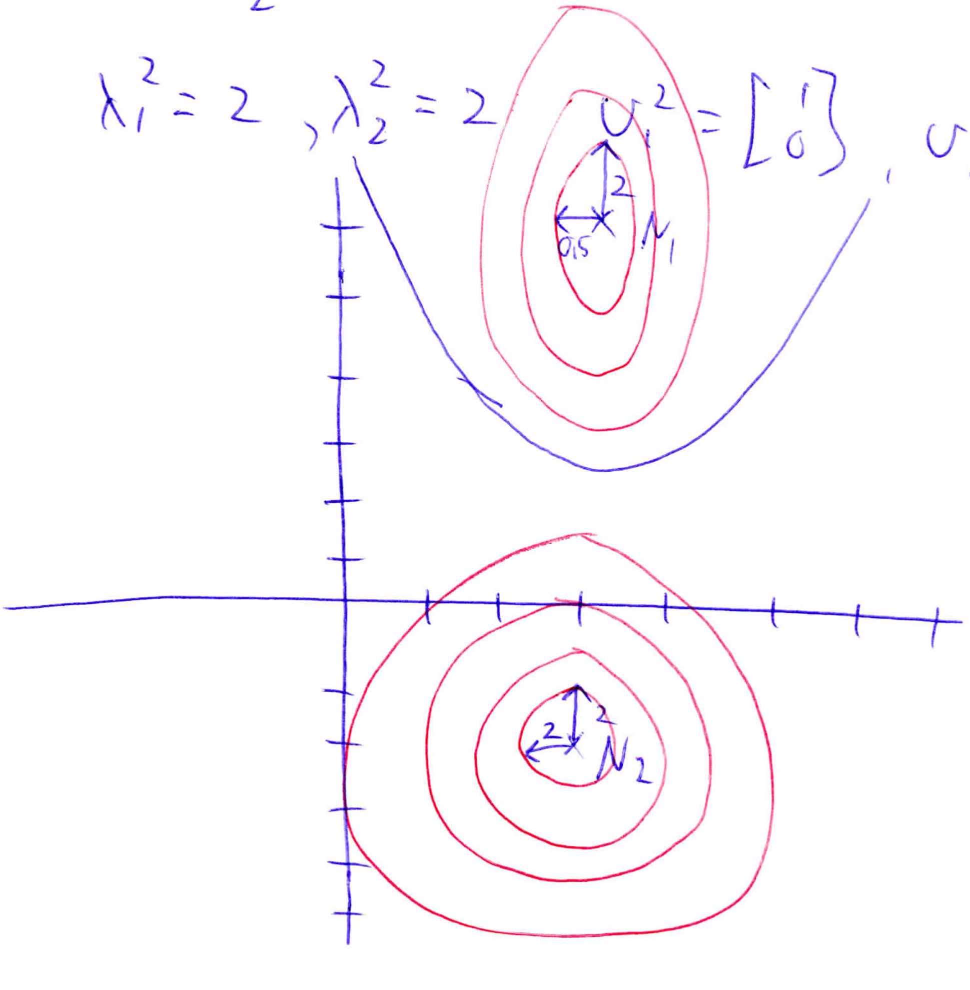

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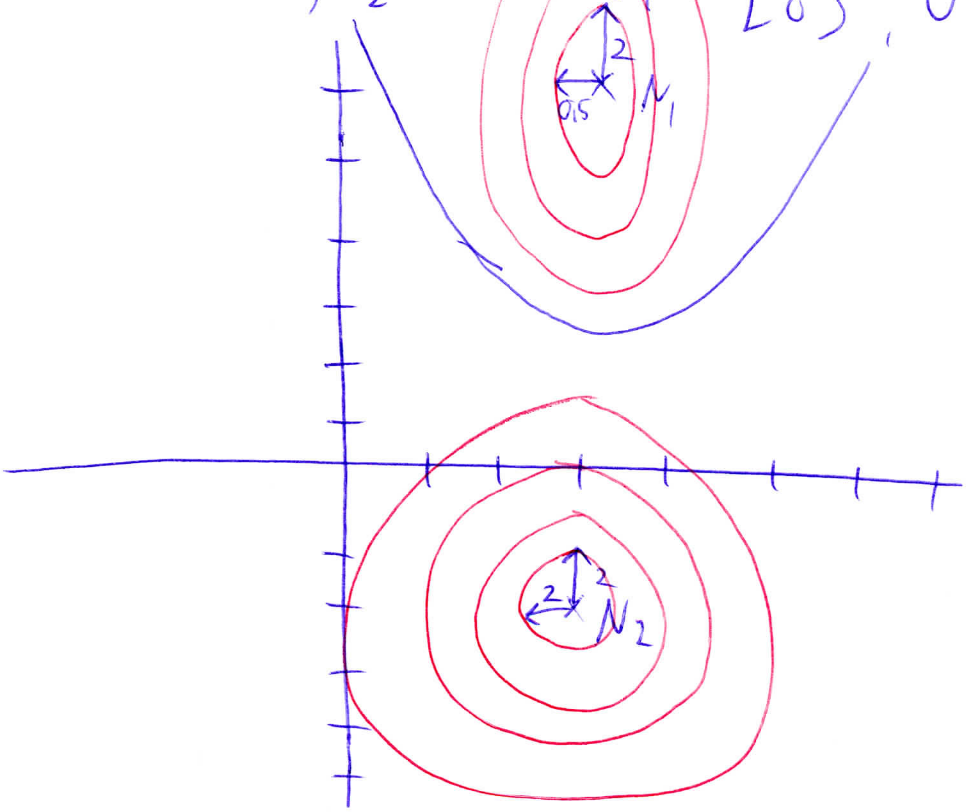

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$$= -\frac{1}{2}\begin{bmatrix} 2x_1-6 & \frac{x_2}{2}-3 \end{bmatrix} \begin{bmatrix} x_1-3 \\ x_2-6 \end{bmatrix}$$

$$= -\frac{1}{2}\left(2x_1^2 - 6x_1 - 6x_1 + 18 + \frac{x_2^2}{2} - 3x_2 - 3x_2 + 18\right)$$

$$= -x_1^2 + 6x_1 - \frac{x_2^2}{4} + 3x_2 - 18$$

$$g_2(\bar{x}) = -\frac{1}{2}\begin{bmatrix} x_1-3 & x_2+2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1-3 \\ x_2+2 \end{bmatrix} - \frac{1}{2}\ln 4$$

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$$= -\frac{1}{2}\left(\frac{x_1^2}{2} - \frac{3}{2}x_1 - \frac{3}{2}x_1 + \frac{9}{2} + \frac{x_2^2}{2} + x_2 + x_2 + 2\right) - \frac{1}{2}\ln 4$$

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So $g_1(x) = g_2(x)$ is:

$$4x_2 = \frac{3}{4}x_1^2 - \frac{9}{2}x_1 + \frac{59}{4} - \frac{1}{2}\ln 4$$

$$x_2 = \frac{3}{16}x_1^2 - \frac{9}{8}x_1 + \frac{59}{16} - \frac{1}{8}\ln 4$$

$$x_2 = 0{,}1875 x_1^2 - 1{,}125 x_1 + 3{,}5142$$

QED.

Classification 3

Ex 1a)

$$\Sigma_1 = \begin{bmatrix} 0{,}5 & 0 \\ 0 & 2 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\mu_1 = [3 \ \ 6]^T \qquad \mu_2 = [3 \ \ -2]^T$$

Finding the eigenvalues of Σ_1:

$$\det \begin{bmatrix} 0{,}5-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} = (0{,}5-\lambda)(2-\lambda)$$

So the eigenvalues is $\lambda_1' = 0{,}5$, $\lambda_2' = 2$.

And the eigenvectors must be $v_1' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v_2' \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

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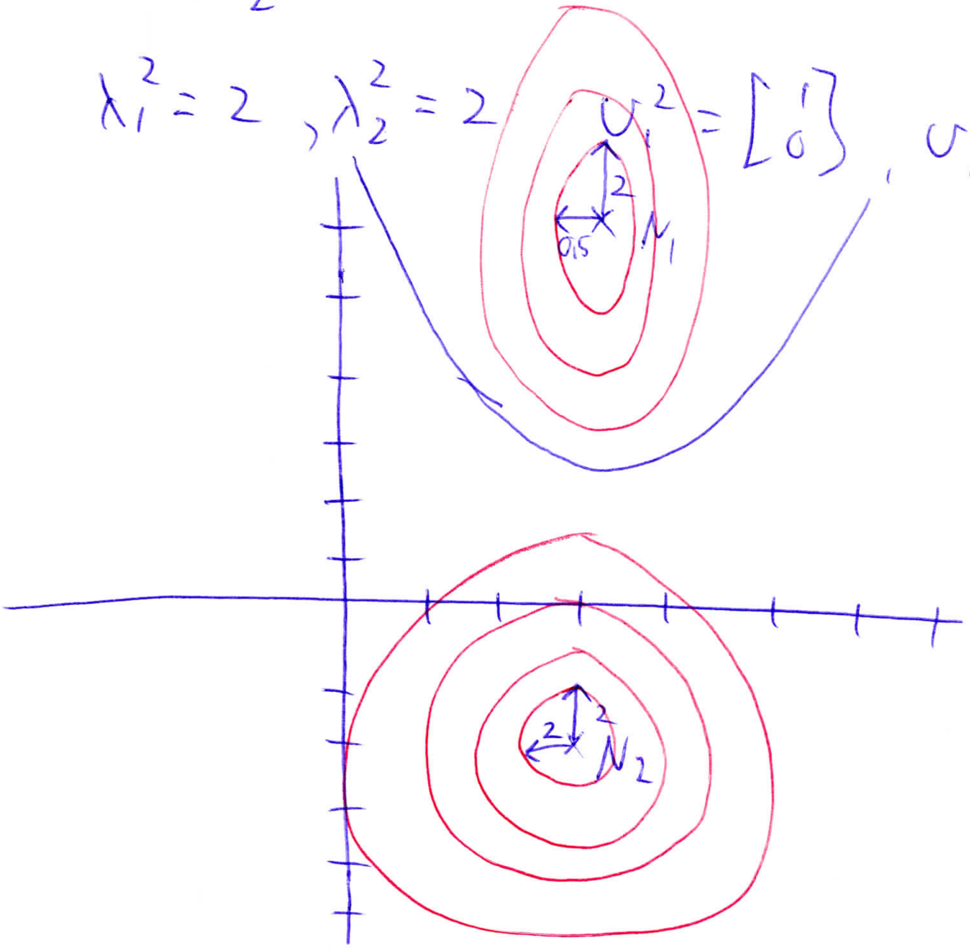

Classification 3

Ex 1 b) Show that $g_1(x) = g_2(x)$ in the case with features x_1 and x_2 can be expressed as:

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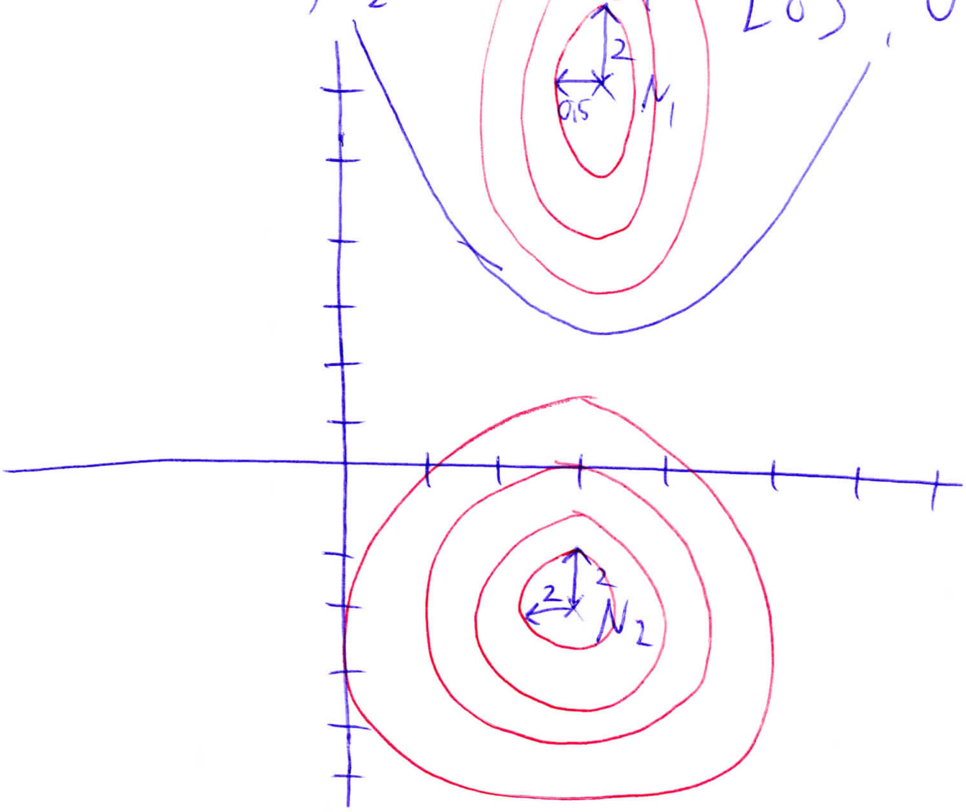

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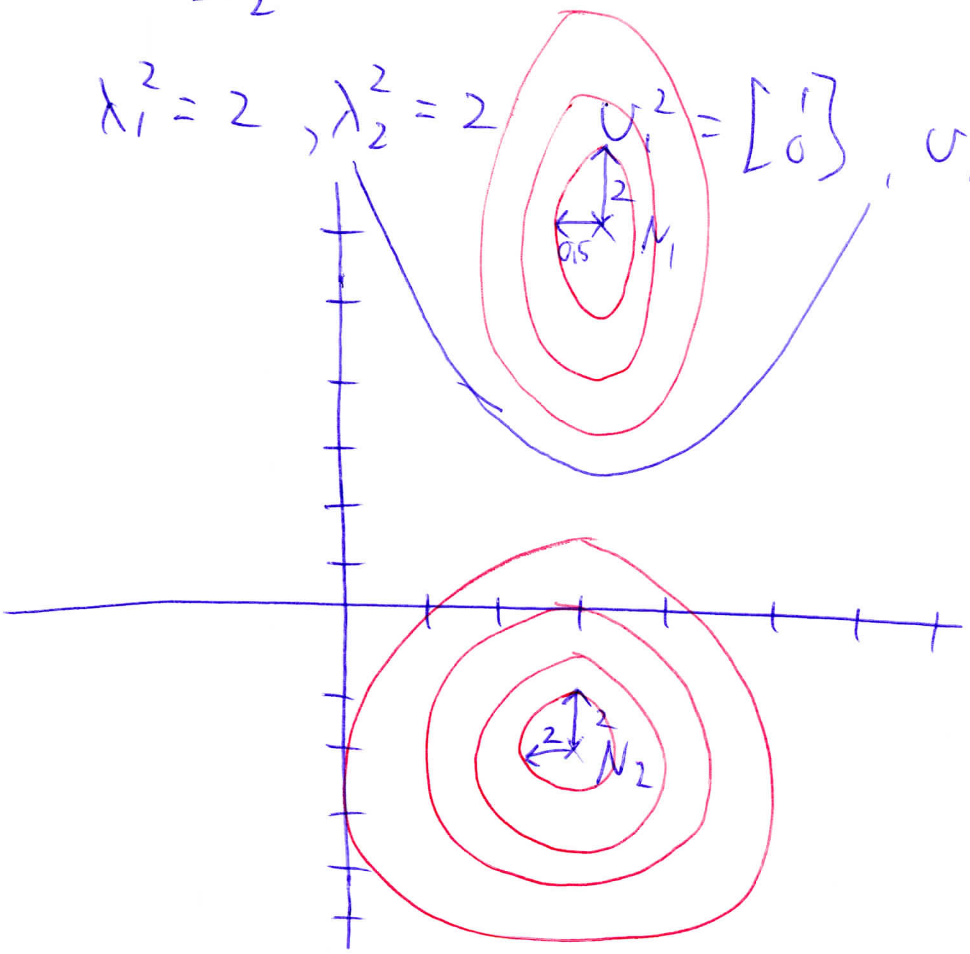

Classification 3

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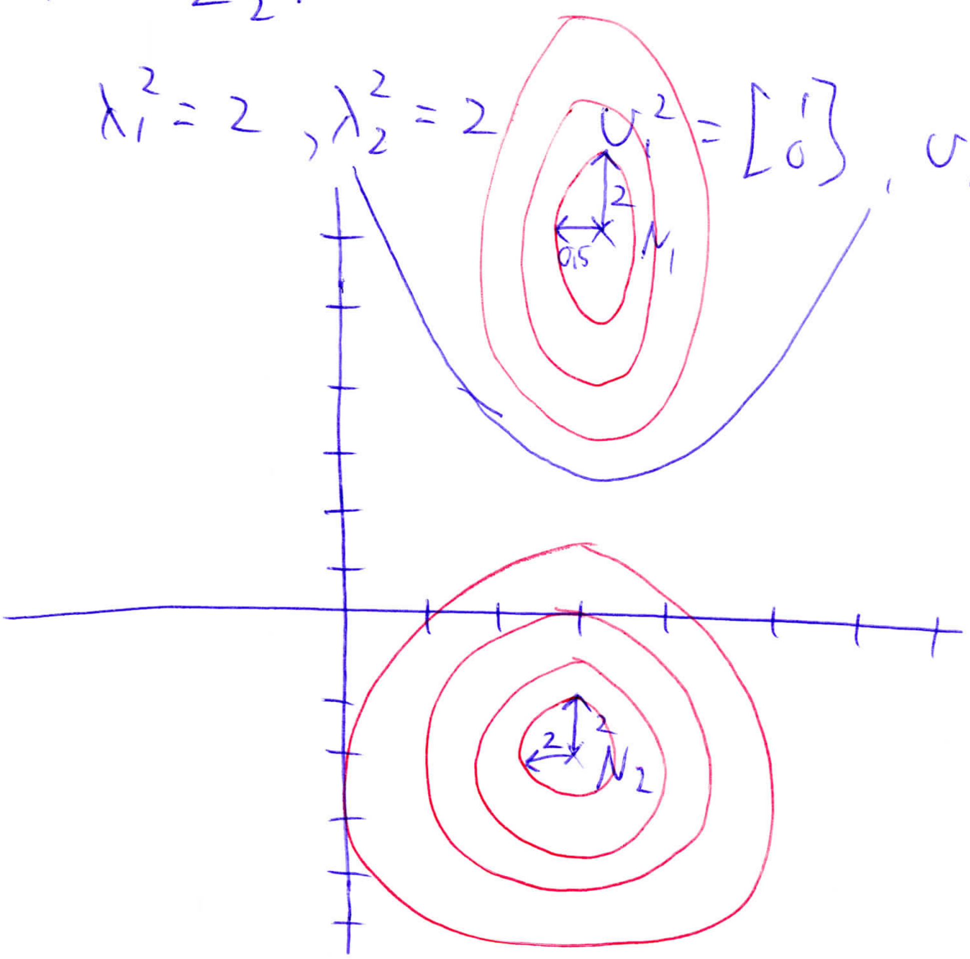

Classification 3

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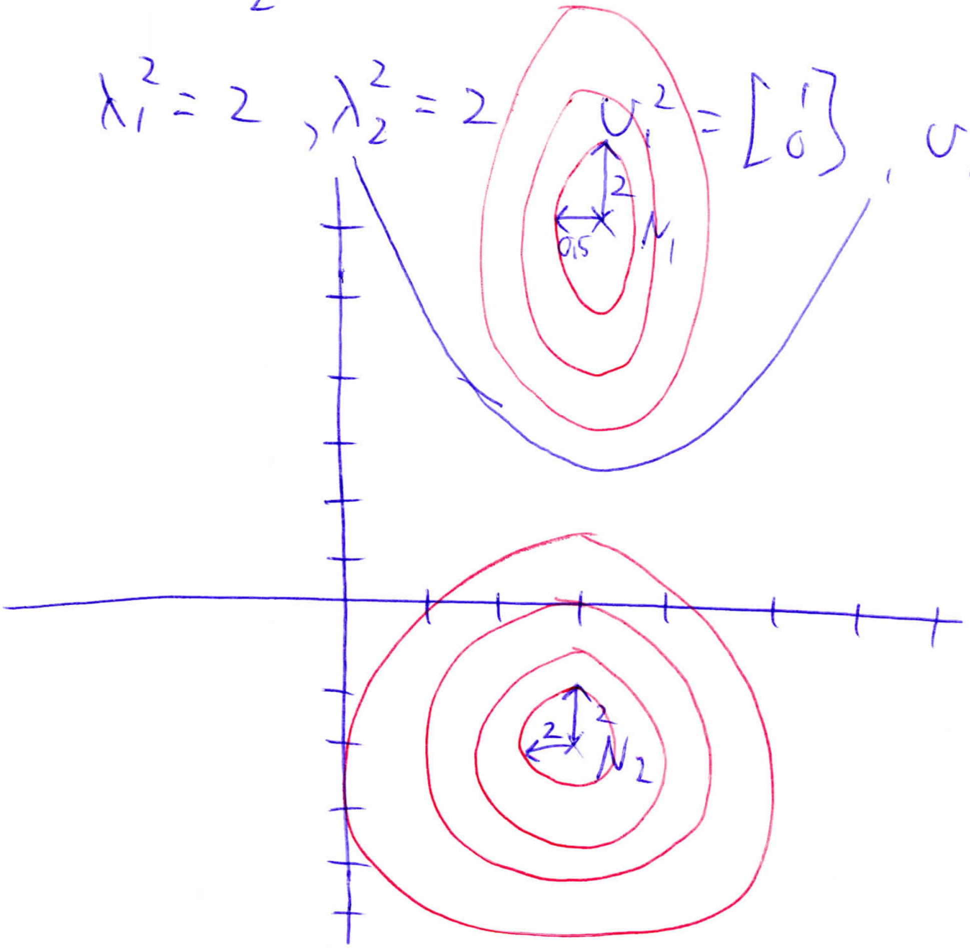

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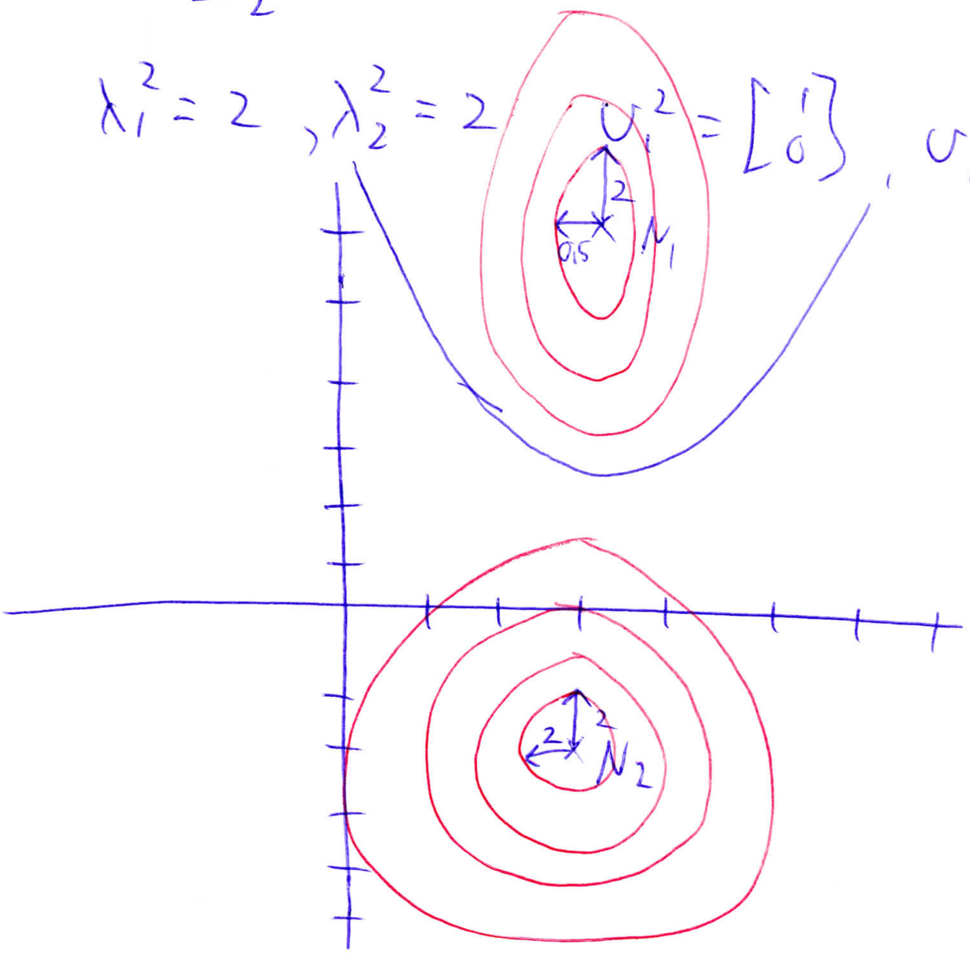

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$$= -\frac{1}{2}\left(\frac{x_1^2}{2} - \frac{3}{2}x_1 - \frac{3}{2}x_1 + \frac{9}{2} + \frac{x_2^2}{2} + x_2 + x_2 + 2\right) - \frac{1}{2}\ln 4$$

$$= -\frac{x_1^2}{4} + \frac{3}{2}x_1 - \frac{x_2^2}{4} - x_2 - \frac{13}{4} - \frac{1}{2}\ln 4$$

So $g_1(x) = g_2(x)$ is:

$$4x_2 = \frac{3}{4}x_1^2 - \frac{9}{2}x_1 + \frac{59}{4} - \frac{1}{2}\ln 4$$

$$x_2 = \frac{3}{16}x_1^2 - \frac{9}{8}x_1 + \frac{59}{16} - \frac{1}{8}\ln 4$$

$$x_2 = 0{,}1875x_1^2 - 1{,}125x_1 + 3{,}5142$$

QED.

Classification 3

Ex 1a)

$$\Sigma_1 = \begin{bmatrix} 0,5 & 0 \\ 0 & 2 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\mu_1 = [3 \ \ 6]^T \qquad \mu_2 = [3 \ \ -2]^T$$

Finding the eigenvalues of Σ_1:

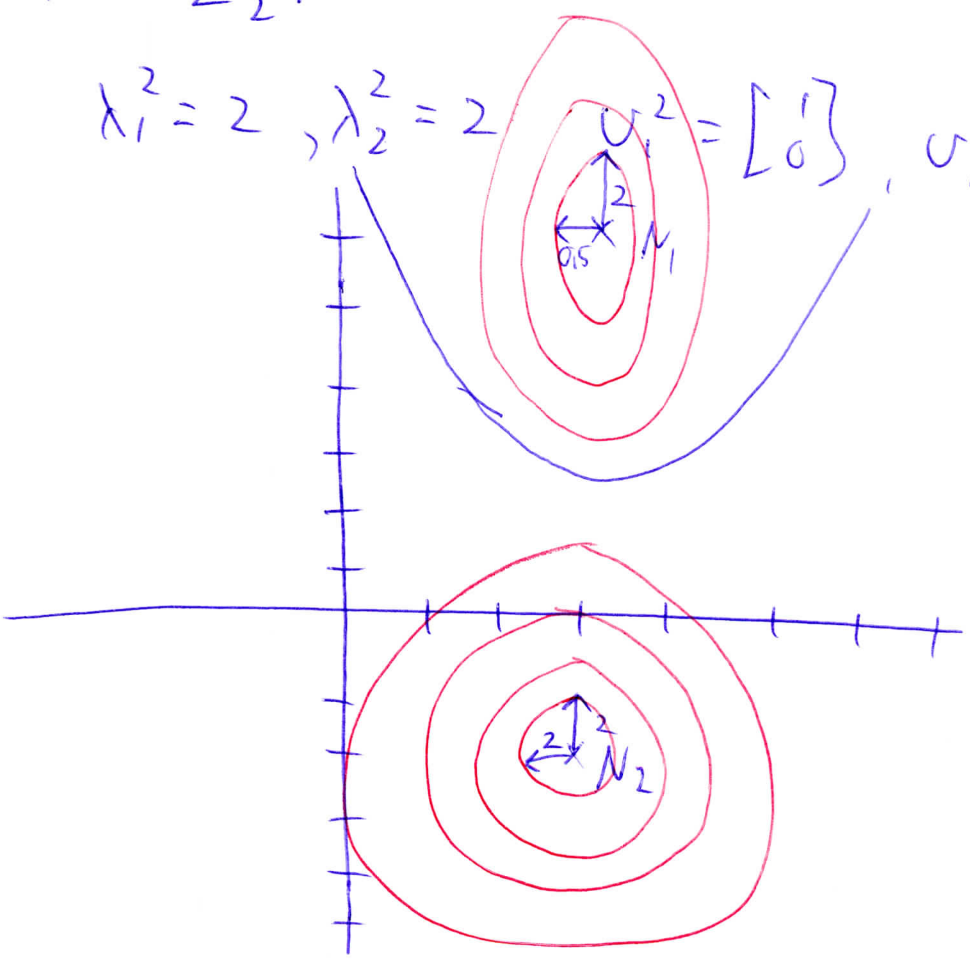
$$\det \begin{bmatrix} 0,5-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} = (0,5-\lambda)(2-\lambda)$$

So the eigenvalues is $\lambda_1^1 = 0,5$, $\lambda_2^1 = 2$.

And the eigenvectors must be $v_1^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v_2^1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

For Σ_2:

$\lambda_1^2 = 2$, $\lambda_2^2 = 2$, $v_1^2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v_1^2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



Classification 3

Ex 1 b) Show that $g_1(x) = g_2(x)$ in the case with features x_1 and x_2 can be expressed as:

$$x_2 = 0{,}1875x_1^2 - 1{,}125x_1 + 3{,}5142$$

We have the general case, so $g_j(\bar{x}) = -\frac{1}{2}(\bar{x}-\bar{\mu}_j)^T \Sigma_j^{-1}(\bar{x}-\mu_j) - \frac{1}{2}\ln|\Sigma| + \underbrace{\ln P_j}_{0}$

$$g_1(\bar{x}) = -\frac{1}{2}\begin{bmatrix} x_1-3 & x_2-6 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 0{,}5 \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2-6 \end{bmatrix} - \frac{1}{2}\ln 1$$

$$= -\frac{1}{2}\begin{bmatrix} 2x_1-6 & \frac{x_2}{2}-3 \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2-6 \end{bmatrix}$$

$$= -\frac{1}{2}\left(2x_1^2 - 6x_1 - 6x_1 + 18 + \frac{x_2^2}{2} - 3x_2 - 3x_2 + 18\right)$$

$$= -x_1^2 + 6x_1 - \frac{x_2^2}{4} + 3x_2 - 18$$

$$g_2(\bar{x}) = -\frac{1}{2}\begin{bmatrix} x_1-3 & x_2+2 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2+2 \end{bmatrix} - \frac{1}{2}\ln 4$$

$$= -\frac{1}{2}\begin{bmatrix} \frac{x_1}{2}-\frac{3}{2} & \frac{x_2}{2}+1 \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2+2 \end{bmatrix} - \frac{1}{2}\ln 4$$

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$$x_2 = 0{,}1875x_1^2 - 1{,}125x_1 + 3{,}5142$$ QED.

Classification 3

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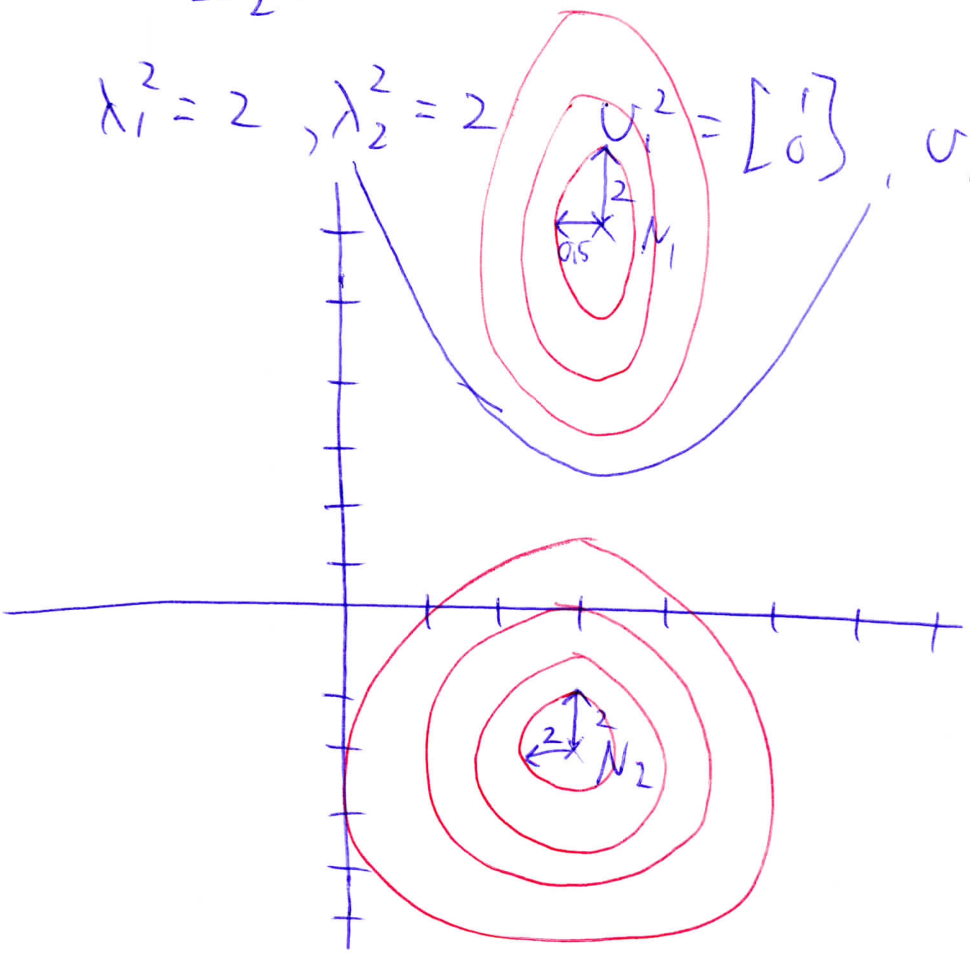

Classification 3

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$$g_1(\bar{x}) = -\frac{1}{2}\begin{bmatrix} x_1-3 & x_2-6 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 0{,}5 \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2-6 \end{bmatrix} - \frac{1}{2}\ln 1$$

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$$= -x_1^2 + 6x_1 - \frac{x_2^2}{4} + 3x_2 - 18$$

$$g_2(\bar{x}) = -\frac{1}{2}\begin{bmatrix} x_1-3 & x_2+2 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2+2 \end{bmatrix} - \frac{1}{2}\ln 4$$

$$= -\frac{1}{2}\begin{bmatrix} \frac{x_1}{2}-\frac{3}{2} & \frac{x_2}{2}+1 \end{bmatrix}\begin{bmatrix} x_1-3 \\ x_2+2 \end{bmatrix} - \frac{1}{2}\ln 4$$

$$= -\frac{1}{2}\left(\frac{x_1^2}{2} - \frac{3}{2}x_1 - \frac{3}{2}x_1 + \frac{9}{2} + \frac{x_2^2}{2} + x_2 + x_2 + 2\right) - \frac{1}{2}\ln 4$$

$$= -\frac{x_1^2}{4} + \frac{3}{2}x_1 - \frac{x_2^2}{4} - x_2 - \frac{13}{4} - \frac{1}{2}\ln 4$$

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QED.