Suggested solution for the 2013 exam

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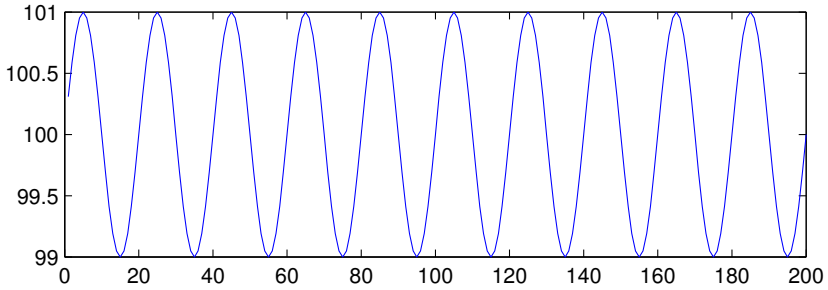
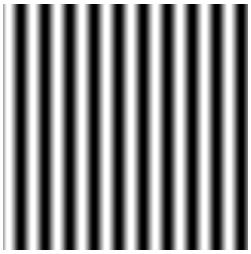
Exercise 1: Texture

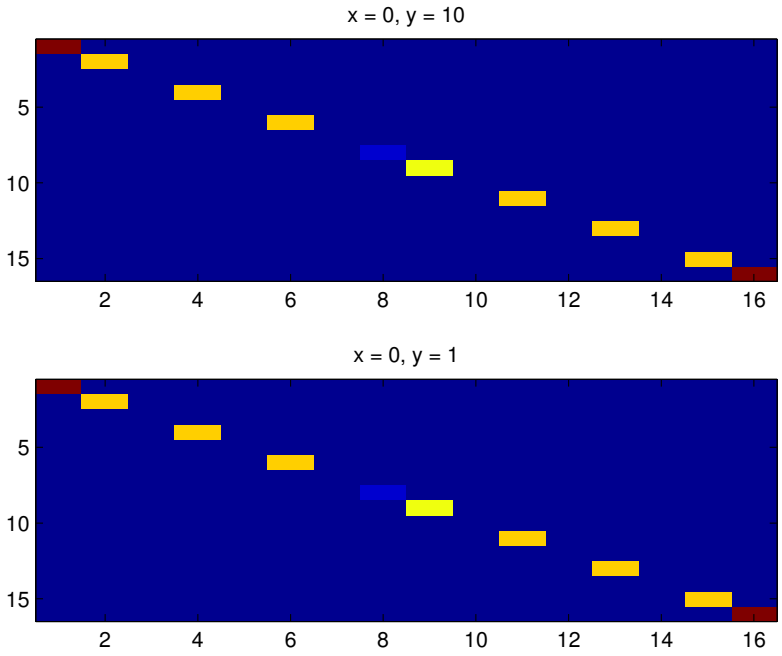




a)

The shape of the GLCM will be points on the diagonal $i = j$. Since the offset in the x direction is set to zero, we will only change from one graylevel to the same graylevel.

A different offset of Δy if $\Delta x = 0$ will not change the GLCM because we will still only change from one graylevel to the same graylevel.

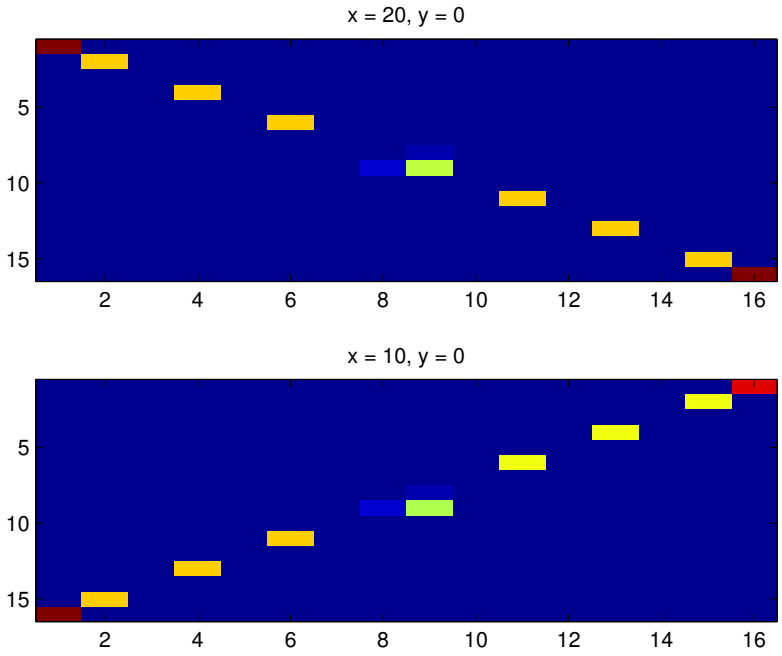


b)

If we choose $\Delta y = 0$ and $\Delta x = 20$ we will hit exatly the period of the signal and thus the GLCM will only have points on the diagonal $i = j$ because we are still just going from one graylevel to the same graylevel.

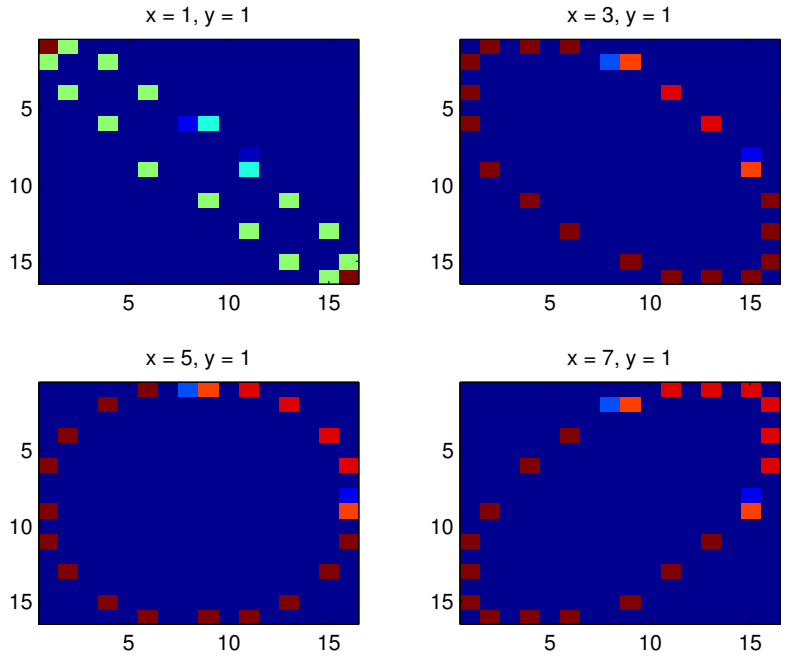
c)

If we choose $\Delta y = 0$ and $\Delta x = 10$ we will hit half the period of the signal. This will give us a GLCM with values on the off-diagonal since we will hit pixels with the "opposite" value of the first pixels. For example when we are at a pixel with value 1 we will hit a pixel with value 16, when we are at a pixel with value 8, we will hit a pixel with value 8, and so on.



d)

The GLCM will start of as a diagonal when $\Delta x = 0$ and as we increase Δx it will become a ellipse and be a circle when $\Delta x = 5$, then it will continue as a ellipse in the opposite direction and finally become points on the off diagonal when $\Delta x = 10$.



Exercise 2: Texture

a)

The image contains a small range of graylevels, only graylevels around 16 are present (13-18).

b)

This is a isotropic GLCM, so since the image contains only similar gray levels in all directions, the image is fairly homogeneous with gray levels around 16.

c)

This is said to be a tricky question. First, we need to know that all probabilities in the normalized GLCM lies between 0 and 1 and sum to 1.

Secondly the weight function of the two features belong to two different categories: the weight function in the ENT feature depends on the value of each entry in the normalized GLCM, while the IDM feature depends on the position on the value of each entry within the GLCM

The $-log_2(P(i,j))$ has a high value for small values of P and goes to zero for P=1.

The IDM weight function is 1 along the GLCM diagonal and decreases very rapidly away from the diagonal.

For a complete homogeneous 32x32 normalized GLCM each of the 1024 cells would have a probability of $\frac{1}{2}^{10}$ and the ENT value would be 10, while IDM = 0.09.

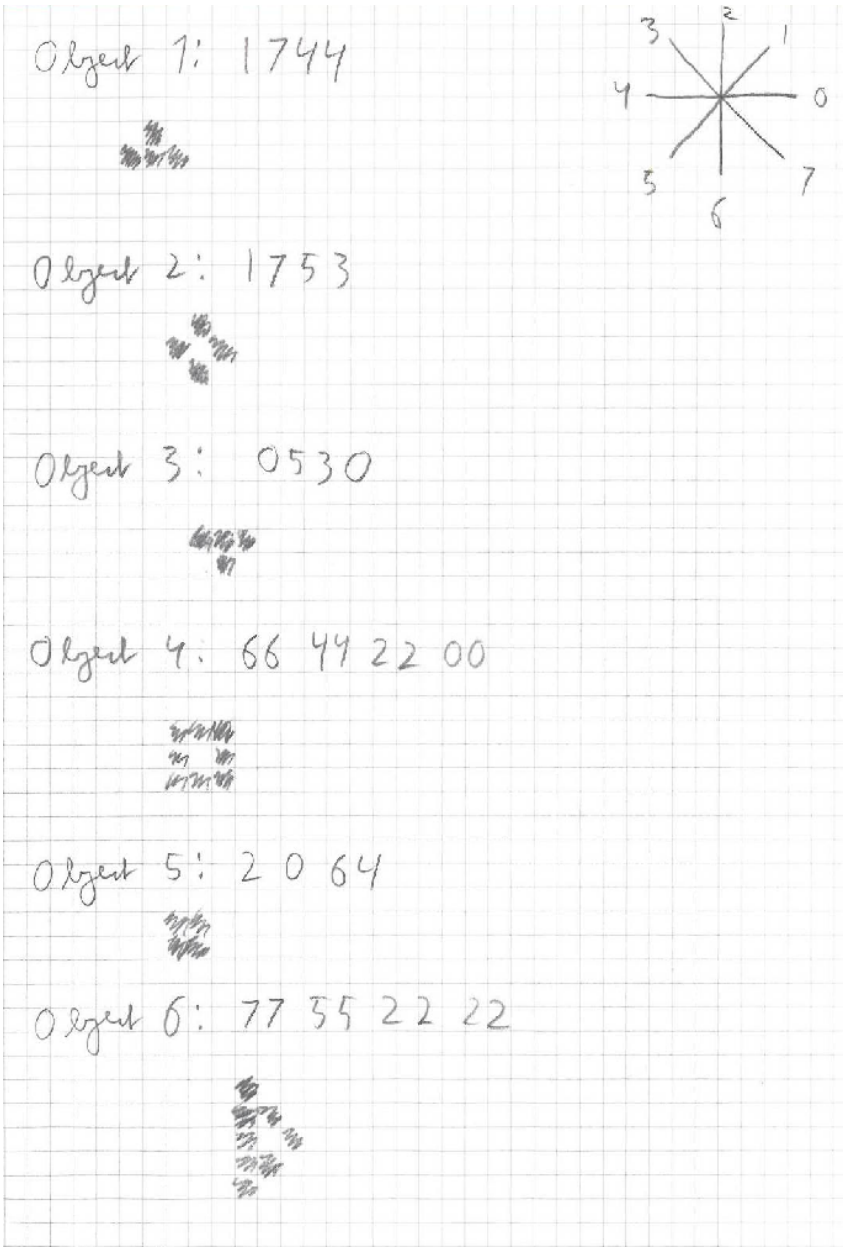
On the other extreme a GLCM having only one non-zero probability on the GLCM diagonal would give IDM = 1 and ENT = 0. Two, equal non zeros elements on the diagonal would gove IDM=ENT=1, while four equal non-zero probabilities on the GLCM diagonal would give IDM = 1 and ENT = 2.

So for our case IDM = 0.6, and ENT = 4.5, and we should be able to somehowe arrive at the conclusion that ENT would get a higher value than IDM.

Exercise 3: Discrimination of objects based on chain codes

a)

Three triangles and three squares of varying rotation and size.



b)

We need to make the chain code invariant to starting point by using minimum magnitude, and make the chain code invariant to rotation by using first difference or relative chain code.

No, because chain code alone cannot be give the same code for a larger and a small objects within the same class.

c)

Many possibilites, for example; only storing the changes in direction, or counting the number of direction changes in the relative chain code.

Exercise 4: Clustering and classification

a)

$$\mu_1 = (7, 5), \mu_2 = (9, 9), \mu_3 = (11, 3)$$
$$\omega_1 = [(7, 5), (7, 7)]$$
$$\omega_2 = [(9, 9), (8, 10), (11, 11), (13, 11)]$$
$$\omega_3 = [(11, 3), (9.3), (13, 5), (14.4), (15, 7)]$$

Recomputing means:

$$\mu_1 = (7, 6), \mu_2 = (10.25, 10.25), \mu_3 = (12.4, 4.4)$$
$$\omega_1 = [(7, 5), (7, 7), (9, 3)]$$
$$\omega_2 = [(9, 9), (8, 10), (11, 11), (13, 11)]$$
$$\omega_3 = [(11, 3), (13, 5), (14, 4), (15, 7)]$$

Recomputing means:

$$\mu_1 = (7.6, 5), \mu_2 = (10.67, 10.67), \mu_3 = (12.4, 4.4)$$
$$\omega_1 = [(7, 5), (7, 7), (9, 3)]$$
$$\omega_2 = [(9, 9), (8, 10), (11, 11), (13, 11)]$$
$$\omega_3 = [(11, 3), (13, 5), (14, 4), (15, 7)]$$

We need two iterations before no point will change cluster during a new iteration.

b)

We have equal diagonal covariance matrices ($\Sigma = \sigma^2 I$), so the decision boundaries will be normal to the line between the means. And since we can assume that the a priori probability is equal the decision boundaries will be half way between the means.

c)

This classifier will classify all training samples correctly, so the error will be 0.

d)

Common covariance matrix means that both classes will have a common elongated ellipse centered at the two new means. If we remove the mean with center at (7,6) both classes will be well described with a commong elongated ellipse centered at the two new mean values. If we remove one of the other clusters, the classes will not be well modelled with a covariance matrix with the same orientation.

Exercise 5: Mathematical morphology

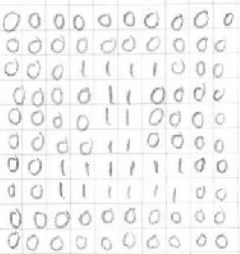
a)

Morphological opening is given by $f \circ s = (f \ominus s) \oplus s$, thus an erosion followed by a dilation. The erosion of an image removes all structures that the structuring elements cannot fit inside, and shrinks all other structures. Dialating the result of the erosion with the same structuring element, the structures that survived the erosion will be restored. This can be used to create an opening between two structures that are connected only by a thin bridge, without shrinking the structures.

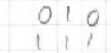
b)

The Top-hat transform is given as $f - (f \circ s) = f - (f \ominus s) \oplus s$. The Top-hat transform gives us the 1's that were removed during the opening since top-hat is the difference between the original image and its opening.

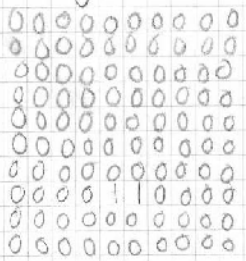
Orig



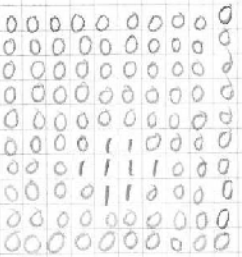
Structure



Erosion $f \ominus S$ Structuring element must fit

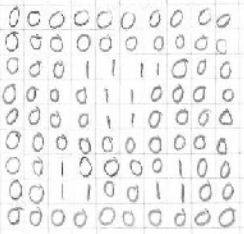


$(f \ominus S) \oplus S$ Dilation: Structuring element must hit

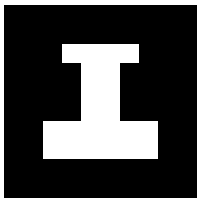


Orig - opening

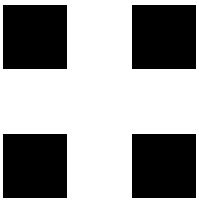
Top-hat: $f - f \circ S = f - (f \ominus S) \oplus S$



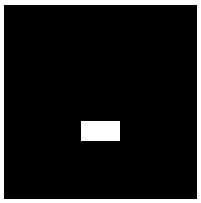
Original image



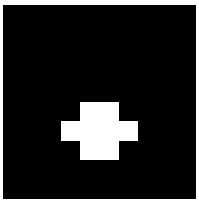
Structuring element



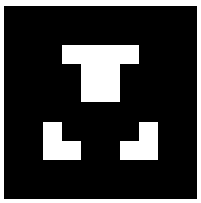
Eroded image



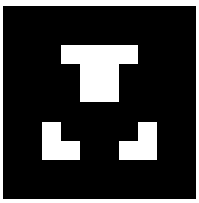
Opening : dialation of eroded image



Original minus opened : top hat



Top hat



Exercise 6: Object features - moments

a)

The moments of intertia are the second order central moments, generally defined as

$$u_{pq} = \sum_x \sum_y (x - \overline{x})^p (y - \overline{y})^q f(x, y).$$

This is the moments around the two axes that are parallel to the image axes x and y, passing through the center of mass of the object ($\overline{x}$ and $\overline{y}$).

$$\mu_{20} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (x - \overline{x})^2 f(x, y)$$
$$\mu_{02} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (y - \overline{y})^2 f(x, y)$$
$$\mu_{11} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (x - \overline{x})(y - \overline{y}) f(x, y)$$

b)

For an 2D object to exhibit a unique orientation the requirement is simply $\mu_{20} \neq \mu_{02}$.

c)

$$m_{pq} = \sum_x \sum_y x^p y^q f(x, y)$$

First, we need to see that the "same moment of interia about the parallel image coordinate axes (x=0 and y=0)" is

$$m_{20} = \sum_x \sum_y x^2 f(x, y) = \sum_x \sum_y (x - 0)^2 f(x, y)$$
$$m_{02} = \sum_x \sum_y y^2 f(x, y) = \sum_x \sum_y (y - 0)^2 f(x, y)$$

We also need to know that

$$\overline{x} = \frac{m_{10}}{m_{00}}$$
$$\overline{y} = \frac{m_{01}}{m_{00}}$$

Then we can for example look at the second order moment μ_{20}, and find that:

$$\begin{aligned} u_{20} &= \sum_x \sum_y (x - \overline{x})^2 f(x, y) \\ &= \sum_x \sum_y (x^2 - 2x\overline{x} + \overline{x}^2) f(x, y) \\ &= m_{20} - 2\overline{x} m_{10} + \overline{x}^2 m_{00} \\ &= m_{20} - 2\overline{x} m_{10} + \overline{x} \frac{m_{10}}{m_{00}} m_{00} \\ &= m_{20} - \overline{x} m_{10}. \end{aligned}$$

Which is what we wanted to show.

And also for μ_{02}

$$\begin{aligned} u_{02} &= \sum_x \sum_y (y - \overline{y})^2 f(x, y) \\ &= \sum_x \sum_y (y^2 - 2y\overline{y} + \overline{y}^2) f(x, y) \\ &= m_{02} - 2\overline{y} m_{01} + \overline{y}^2 m_{00} \\ &= m_{02} - 2\overline{y} m_{01} + \overline{y} \frac{m_{01}}{m_{00}} m_{00} \\ &= m_{02} - \overline{y} m_{01} \end{aligned}$$